CRITICISMS AND DISCUSSIONS.

LOGIC AS RELATION-LORE.

REJOINDER TO M. MOURET BY MR. RUSSELL.

The strong and tolerant reply of M. Mouret to my criticisms demands from me my best consideration, not only on account of the ability and courtesy he has shown but also on account of the very important matters that are thus agitated.

I must trespass upon his patience still further, for so far am I from being more at one with him than before, that I fear that that community of view which he believed to exist between us, and which I certainly took to be the case, is in fact much less than either of us have been anticipating.

I. THE NATURE OF MATHEMATICS.

M. Mouret, if I judge him aright, is fully persuaded in his own mind that the mathematicians are lacking in philosophical competency, and are doing little if anything towards the right settlement of the fundamental principles and data of even their own branch of science. I, on the contrary, see in the works of the mathematicians, that which leads me to expect from them, or at least to expect in consequence of what they have accomplished and are going on to accomplish, a new and most illustrious phase in the history of philosophy.

M. Mouret tells us explicitly, that he has occupied himself in analysing the fundamental notions of mathematics, in part, by way of protest against certain doctrines that have the countenance of at least many of the French mathematicians. If I may judge from what I confess to be an altogether inadequate acquaintance with the writings to which M. Mouret refers, I should be inclined to say that he fails to do the men in question the complete justice that the dignity and worth of their work merits. They are, however, men of distinguished ability, and any attempt by me on their behalf would be both officious and presumptuous.

I feel moved, nevertheless, to offer a few words in regard to the question as to the meaning and scope of mathematics. The time is long past when mathematics can with any propriety be defined as the science of quantity. Such is the old

stock definition and no doubt the impression is very generally prevalent tha mathematics is naturally confined within the range where quantity is a prominent feature of the things to be dealt with. But if nothing else than projective geometry and the theories of substitutions and of groups were extant, there would be enough to show that no pent-up region like the region of quantity can confine the powers of mathematics. When, however, we take notice of algebras like "Peirce's Linear Associative Algebra," the various logical algebras, and other very possible algebras that these suggest, and especially when we consult the splendid "Theory of Mathematical Form" of Mr. A. B. Kempe we begin to suspect what I dare say is really the truth that mathematics is the imperial science, whose prime data and original principles must govern the entire range of intellectual exercise.

What then is mathematics? How is it to be defined? No competent answer can be given to these questions until we recognise that the soul of mathematics dwells not so much in the terms or things with which it deals, or even in the static relations, that may obtain in respect thereto as in the *operations* by which these terms or things may be put together or separated and by which the same and said static relations may be altered. Pure mathematics does not even require that all its data and results shall be conceivable, that is, that they shall be of such a nature as to excite in us those mental phenomena that we usually mean when we speak of sensations, notions, concepts, or perceptions. The square root of negative unity, and infinity, the curved, and four or more dimensioned spaces, are mathematical things that we ought not to expect to render (readily at least) into terms of ordinary mentality. Much familiarity with them may in time evoke corresponding mental sensations, perceptions, etc., but until these naturally emerge so as to accredit themselves according to their proper significance, we ought to regard these supra-conceptual things as obtaining in *notation* merely. Indeed it may be more than a fanciful manner of speech to say that in mathematical notation there is being evolved a new supra-intellectual faculty for man.

Mathematics is not, however, by any means lawless, nor is it metaphysical in any bad sense of that term. Variable, indeterminate, and incommensurable are indeed many of its things and functions respectively, but vague or inexact not at all. Even the supra-conceptual things above mentioned are as rigidly ruled by the mathematical constitutions as are all the rest. Nor are the same in any proper sense abstractions or compounded of abstractions. They are the suggestions and the results that have ensued in consequence of the nature of the mathematical *operations*. These have pleaded for what was needed in order to enable mathematics to fill itself out so as to occupy its own proper sphere, and hence the supra-conceptuals have been *recognised* and installed.

In this filling-out of the mathematical sphere as well as everywhere else in mathematics one supreme rule obtains, viz., the rule of *consistency*. No contradictions must be *involved*, but however the data and functions of mathematics are interworked, all must harmoniously co-operate and issue. In point of fact they not

merely do this but they relate and operate together so as to form one harmonious and mutually illuminating whole, in sum and by every detail. It is this that makes mathematics the *beau ideal* and great exemplar for all science and all philosophy.

Mathematics is based upon one original tenet, theorem, faith, or supposition, viz., that this universe of matter, energy, and mind is throughout consistent and reasonable or rather consistent (which *is* reasonable) or reasonable (which *is* consistent).

Another article of the mathematical creed, and one scarcely secondary, is this, There is one absolutely *unique* system of principles consisting of divers operations, relations, notions, and recognitions, which system is necessary and sufficient for the general organisation and explanation of the consistent and reasonable *All* : besides which system, and besides each element and detail of which system, there is naught else either sufficient or necessary for such organisation or explanation ; and lacking which system or lacking in certain elements or details of which system, or lacking in some element or detail of which system, such organisation and explanation must proportionately fail. For brevity I will call this system of principles the *organonic* system, and the principles thereof the *organonic* principles.

Now mathematics is that model science or that commonwealth of model sciences, that observes, certifies, and applies the organonic principles, for the improvement of knowledge and belief.

While the question of the genesis of our knowledge of the organonic principles has a decided interest of its own it has little if any relation to their justification. Not how we came by them but what they really *are*, is the important question. Beside the questions as to *what* numerical unity really is, or as to *what* a straight line is, all questions as to how we acquired the notions of these prime data respectively fall into insignificance. We should not discard them even though it should turn out that we acquired them by questionable means. Our title to them is our perception that they naturally belong to us. Just such, too, is our title to the supra-conceptual recognitions. The imaginary unit is known to belong to us not alone by right of adoption, but principally because it exactly fits and fills out the numeric sphere, the other part of which has long been ours. While a knowledge of the genesis of the organonic principles may yield divers suggestions as to the nature thereof, it is nevertheless a knowledge of that nature that we principally need, and this is mainly a matter of observation and criticism.

The recognition and adoption by mathematics of the supra-conceptual entities and the coincident, necessarily implicit transition of the mathematical sovereignty from the passive things with which it deals to its operations, marks, in my judgment, an absolutely new era in mathematics, and through mathematics an absolutely new era in science and philosophy. It stands as a kind of scientific and philosophical Peak of Darien from which we look out on a new ocean for science and philosophy, an ocean palpable and differing in no essential respect from the regions with which we are familiar, but swelling with surges that signify a vastness until now undreamed

of, and yet an ocean for the exploration of which we are, at least in embryo, already furnished.

Here several reflexions push forward. The first is the sovereignty and exigency of the demand for consistency, completeness, unity. Monism is inevitable because nothing less is competent to effect any settlement that will actually settle. Hitherto it has been taken as a prime supposition, and one taken to be so obvious and insupera- able that it has never even occurred to any one to challenge the same, that the manifold of *conceivable* things exhausted the data of science and philosophy, and would, could only its hidden organic scheme be once discovered, stand revealed as the all in all. In demand, however, for a unity, a consistency, a wholeness, that could not, nor would, otherwise emerge, and being moreover, as if in duty bound, in obedience to principles long approved as valid and in the highest degree fruitful, to appropriate, in some way, in its scheme certain somewhats that could not be ban- ished, but were ever thrusting themselves forward in its very face and eyes, math- ematics at last broke over the charmed circle of conceivable things, recognised and adopted as its own property the so-called *imaginary* data, and instead of finding itself put to absurdity and confusion, found itself, on the contrary, unshackled merely within a renewed, but immensely expanded, world of rigidly consistent verities, that contains the old world as a fragment.

This epoch in the history of mathematics, when its nature is duly appreciated, gives a most momentous lesson to science and philosophy. In philosophy especially we can no longer account the manifold of conceivable things as the all in all, and confine the exercise of our philosophical propensities within its range. We can no longer entertain any rational expectation of finding therein that consistency, com- pleteness, and unity of which we are in search. We can now no more do this than could the old world, after the successful voyages of Columbus and his followers, account the mere eastern continent to be the entire earth, and expect to gain a full and competent knowledge of the earth by studying the geography of that continent only.

The second reflexion that pushes forward is that the true justifications of any sort of dialectic are to be found not so much as we have been wont to think, in the conceivability of things or even in the agreement of thought with things, unless that criterion is better understood and applied than it usually is. True thought does and must agree with things because true thought is only the expression, total or par- tial, of the organonic principles which pervade thought and things alike. Things, *when truly and adequately interpreted*, exemplify the organonic principles and suggest them to thought. But things may be badly or inadequately interpreted, and yet, owing to their fragmentary nature, there may be a thought, or system of thought, partial or untrue, that will accord with such interpretation. In short, while incon- ceivability may possibly be an *argument* against a certain thought or quasi-thought, and while the agreement of any assigned thought with things is certainly an argu- ment in favor of the truth of that thought, this inconceivability and this agreement

are both of them uncertain in their significance. There is nowhere any other test of universal application and discriminating authority but that of *general mutual consistency*. This is at once necessary and sufficient to accredit and justify any thought, however inconceivable it may be. If it is said that a thought may, at some one time fully satisfy this criterion so far as can be ascertained, and yet, upon better information, fail therein, I reply, verily such may be the case, but in such a case, however much that thought may swerve from the truth, it will nevertheless mark out and revolve around a verity whose subsequent more exact delimitation it is that reveals the untrue aspect. The criterion laid down by Mr. Spencer, the inconceivability of the contrary, is only one way of stating the criterion of *general mutual consistency*, and is itself subject to the same qualification that has just been stated. The defect of Mr. Spencer's formula dwells in this, that in all that class of cases, by no means rare, in which we can conceive a contrary, we are left without a criterion, unless we say that those ideas are untrue whose contraries are conceivable, a conclusion that is obviously unjustified.

M. Mouret says that the mathematicians with whom he disagrees and whose doctrines served in part as the exciting cause of his essays, "consider mathematics as the science of combinations having for points of departure certain conventions made with numbers independent of reality in general and of the physical magnitudes in particular." If they really do this, they are certainly very loose in their habits of thought. But mathematicians are usually not thus given to looseness of *thinking*, however much they may scatter in their forms of expression. In such recondite regions of discourse as are those in view, there is great need of taking one not exactly and literally as he says, but according to his true intent and meaning, as gathered from his entire context. Says Challis :

" In a passage full of acuteness and good sense, Berkeley ['Theory of Vision,' § cxx] remarks how ill common language is adapted to be the vehicle of uncommon thought, and demands most reasonably that the reader shall strive to follow the thread of his ideas, rather than carp at his language and catch at hitches which the circumstances make inevitable."

I have before me an article by that illustrious French mathematician, M. Poincaré, whom, I take it, is a typical representative of the school of mathematicians that M. Mouret finds so poor in philosophy. In this article, M. Poincaré discourses of the Non-Euclidean Geometry and of axioms, and he does indeed speak of the axioms of geometry as conventions, or as *definitions in disguise*.

The truth is, as it appears to me, somewhat like this : There are real things which may be corporeal things, or relations, or operations ; and there are *meanings*, or notions, or ideas ; a part of which, at least, may represent real things, and are indeed themselves in a certain sense real ; and there are *terms* which are intended to represent, firstly *meanings* and then, in some cases, through meanings, various real things that are not in themselves meanings merely.

Now, besides the various infirmities of terms in themselves, they may be put

together in any way that the mere rules of language permit, and when they are so put together as to form an assertion, there are in any case, two questions that may always arise ; first, Is there any meaning expressed ? and then, Is this meaning, if any, true or otherwise ? In case any assertion is a definition or an attempted defini- tion, it may express, or attempt to express, a status or an operation, or it may sup- pose or quasi-suppose such a status or operation.   In such a case the further very important question may arise.   Is this status or this operation one that is possible or *compassable*?   If not, then the assertion, supposition, or quasi-supposition has really *no* proper meaning, although unfortunately in too many cases a lack of infor- mation or a vague unmathematical habit of mind lures many into the belief that a meaning is contained therein.

But definitions depend upon terms, and by no device of man can they free *meaning* from the trammels and limitations thus entailed, so long as we insist on confining ourselves to the express import of those terms.   So insisting, try how we may, we must always at last resolve on some term or terms that shall be taken as known without more ado.   Are we, then, tied down to the express import of terms ? By no means, else poet and seer must always have gone to their graves undelivered of their burdens so precious to man in all generations.   Signs and sign-systems of all sorts have, besides their express import, a *suggestive* power and function, and no signs or sign-systems have, for this behalf, so efficient a power and function, as have terms and language.   How often even in the daily intercourse of man does he impart his meaning surely and exactly, nay, even more efficiently, by language that by its express import does not mean as he means, but which frequently, in its literal im- port, expresses the precise opposite of his real meaning.   So in science, even in mathematics.   There was never so bald a paradox, according to the express import of the terms used, as to say, "a point is a place without any size whatever," yet it tells the true intent and meaning without any reasonable cause for exception.   It is *meanings*, then, that we are after, and terms and language are only so many con- venient means towards that end.   So far as the axioms and the mathematical points of departure relate to terms and language, they are and must be conventional.

Now, meanings are primarily matters of mental *status*, and, if as to any por- tions of discourse those who discourse together have the same meanings in the same connexions, or experience mental states that obtain the same in each, in a one-to-one correspondence as to the articulated parts, and in a general correspon- dence, as to the wholes, then to cavil at calling this a matter of convention, seems to me rather an exercise of logomachy than otherwise.

The truth really is that many of those propositions that are called axioms and many of the mathematical points of departure, are not axiomatical at all.   Some of them admit of proof when once the real axioms are ascertained and once the requi- site definitions are duly certified.   Taking equality as the conjunction of "not more than" and "not less than," the so-called axiom : "Things that are equal to the same thing are equal to one another," may be proved thus.

Put A and B both equal to C. Then A and B are also equal. For A is "not more than" C, and hence C is "not less than" A. A is also "not less than" C, and hence C is "not more than" A. By precisely parallel reasoning, C may be proved to be both "not more than" and "not less than" B. Then A is "not more than" C, which is "not more than" B; hence A is "not more than" B. So, too, A is "not less than" C, which is "not less than" B, and hence, again, A is "not less than" B. A is thus shown to be at once "not more than" and "not less than" B. But to be these is to be equal to B. *Q. E. D.*

There are other so-called axioms that are, as M. Poincaré says, merely definitions in disguise. Such is this one, "The whole is more than any part." Another one that may easily be taken to be very like the last one but which in reality is quite different, is this, "The whole is equal to the sum of all its parts." This is a proposition admitting of a proof that is somewhat lengthy, and that depends upon the definition of "equality," the definitions of "whole" and "part" and the definition of "sum," which definition depends itself upon the prior definition of the operation of addition, the definition of which operation depends again upon the knowledge derived from the actual performance of a problem ; that is, from experience.

As above stated the possibility or *compassability* of certain states or operations may be brought into question. In many cases this can be determined affirmatively by actually effecting the state or operation in question. Then we have a problem solved and the corresponding knowledge gained by observation or experiment or whatever it may be called. At any rate by experience.

In other cases divers conditions and circumstances may prevent us from actually effecting the state or operation in question. We cannot go to infinity to try the experiment and see whether we can draw a line that will at once meet another line there and also at some finite point. In another class of cases we have no means of certifying what is the real state of the case. We cannot travel all over space so as to test it, and see if it is everywhere all alike. In still another class of cases we have no means of certifying what may be the real effect of certain of our operations. We cannot tell for certain whether the mere moving of a body around in space does or does not alter it in size or in shape. In all these cases of uncertainty our only resource is to resolve on what we will take to be true, and for the sake of the great convenience this course affords, abide by our resolutions ; at least so long as they entail no inconsistency, or contradiction. In order, however, for this course to prove convenient we must agree together expressly or tacitly as to the meanings the propositions shall bear that formulate our resolutions. Such propositions are the only real axioms. All others rest at last upon experience or definitions.

If any one says that such conventions as these are "independent of reality in general," or, if they relate to the physical magnitudes, that they are independent thereof, then if by "independent" is meant independence in any other sense than that that reality or that the physical magnitudes, are silent in respect to the validity

of such conventions, such persons are surely, unqualifiedly and totally mistaken and deserve even more than the reproaches that M. Mouret has heaped upon them.

II. THE NATURE OF RELATIONS.

After all this long dissertation M. Mouret will surely have excuse to ask how the same is any rejoinder to his reply. I must maintain, however, that it is a rejoinder to the most serious parts of his reply, though made by general rather than by particular remarks.

I cannot admit that I failed to understand him in the main. The root of our disagreement lies in our different views of the nature of relations. As long as this difference remains I do not see how we can arrive at any community of view that would be worth any pains to effect. I said in my paper :

" Now besides the error of confounding relations with relationship it is a very common fault to think and speak of relations as being *between* two or more terms. This imports into thought the thoroughly misleading idea of an intervening independent existence for relations. Relations are attributive predicates of terms, and each one of them pertains strictly to its proper term or combination of terms, in the same sense for this turn (*pro hac vice*) that qualities are held to pertain to their so-called substances," etc.

This conception of relations together with my protest against the common view is almost ignored by M. Mouret. I grant that he conforms himself to the ordinary language as well as to that use of the same that is current in the so-called philosophy ; but as a student for a scientific logic, it was on the very account like to that, that I had occasion to make the caution above quoted, and I am not a little nonplussed to find M. Mouret treating the matter in the way he does. If I were the first to discern and employ this notion of a relation, I should, in face of the conduct of M. Mouret, feel great doubt lest I had gone wrong. But since this way of conceiving relations appears to me entirely plain as well as absolutely requisite to a scientific treatment thereof, and since, moreover, in this I only follow the example of DeMorgan and Peirce, and in so far as I am aware, with the single exception of M. Mouret, all the rest of those who in modern times have given any considerable attention to the subject of relations, I naturally feel entirely sure of my ground.

This matter is so fundamental that it calls for defence. Relations, of course, may like other things be of divers sorts ; but in so far as they are relations merely, they must be essentially alike. Hence the nature and characteristics that pertain to relations in their most naked estate must continue to pertain thereto in their every form.

I have to say in this connexion that my interest in the views of M. Mouret was mainly enlisted by his conception of logic. I either found in or read into his language a statement of certain somewhat inchoate ideas about fundamentals that had for a long time been ever and anon flitting athwart my mental horizon.

The title "logic" as it is usually employed is made to comprise several very sep-

arable fields of discourse. If such a separation were made and I had the choice on
that behalf, I should, for reasons not relevant here, incline to give the old title
"logic" to that division that deals with the phenomena of erroneous thinking and its
correction, that is, to the doctrine of fallacies. But in that case logic could have no
claim to the dignity of being the *scientia scientarum*. Before reasoning, good or bad,
can have any occasion to be, the terms and relations upon which it operates must
already obtain, and the operations that marshal the same into various arrays and that
modify those terms, relations, and arrays in various ways must become extant. Hence
the need of an "abstract and objective science that has for its domain the sum total of
the exterior objects of knowledge considered independent of their particular nature."
I took M. Mouret to mean by "exterior objects" not merely corporeal objects, but
everything that bears the insignia of reality, every *fact*, no matter what its nature
may be. These, when "*considered independent of their particular nature*," are
stripped of every vestige of determination and stand nakedly as so many mere some-
things all exactly alike, different indeed as instances but indifferent in all other
respects. Corporeal objects, time and all its events, space and all its configurations,
numbers, orders, arrays, motions, forces, institutions, etc., etc., and all their evolu-
tions and involutions, etc., etc., remain as a plurality of indifferent instances only.

Now, "can these dry bones move?" Yea, verily. Let but that moving spirit
that takes on so many phases, viz., distinction, sundering, denial, etc., etc., and in
virtue of which these things "*independent of their particular nature*," obtain as dis-
tinct instances, and also the antithesis of that moving spirit, which antithesis also
takes on the corresponding antithetical phases, viz., sameness, gathering, affirmation,
etc., etc., in virtue of which the said things "*independent of their particular nature*"
obtain as copies of one another; let but these continue to operate and subsist, and
"form" will evolve in endless luxuriance. Those apparently dependent but really
fundamental things that are relations, will at every stage appear as if newly born,
and out of this fourfold root, to wit : the original of distinction, etc., the original of
sameness, etc., the sense of relation, and the sense of "form" will grow sufficient
reasons whereby to explain all experience.

This "form" and its components generate in us certain psychological effects
which are not merely full of interest on their own account, but *with their suggestions*
form the very occasions for us to perceive "form" and its components.

But these psychological effects, except for the form that may pervade those of
them that are complex, are of no moment for science or philosophy. They are ul-
timate irresolvable facts, and as such only so many "things" to be taken "*inde-
pendent of their particular nature*" like all the rest. *Mere* psychology, that is to
say, psychology less the "form" that it exemplifies, has no instruction to impart
that is of any benefit to science or philosophy. These very psychological effects in
their turn have been created by and depend upon "form." They depend upon that
which we call "mind," or rather that which we call "mind" is the organised (that
is "formed") aggregate of these psychological effects. Mind in its turn depends

upon the organised (that is "formed") aggregation of brain-stuff, and the study of this as well as the study of mind is and can be no other than the study of the "form" exemplified therein, and no other or different in its essential nature from the study of "form" in general.

In short, whatever may be the true nature of the universe, whether taken in its corporeal aspect or in its aspects dynamical or mental, and whatever may be our efforts to comprehend the same in detail or in general, it is amenable to our efforts only as a manifold of "things," mere "things"—some undistinguished and some distinguished from some others. This absence of distinction or presence of distinction is throughout governed by relations, and the distribution of these undistinguished and distinguished things and arrays of things throughout the manifold is modulated by those things we call "laws," giving occasion for us to perceive, study and organise the "forms" that variously obtain.

But the universe, in spite of its segregate nature, is also a unitary whole. This obtains in very virtue that it is throughout pervaded by relations and that it belongs to the very nature of relations thus to connect things together. This is insuperably the case, because, to point out a truth by a paradox, (since such is the only way by which it can be expressed,) the very absence of a relation is itself a relation. There is nothing, nor can there be anything "absolute." Each "thing" is related (usually in many ways) to each other thing in the universe. This is not in virtue of any office that knowing beings fulfil, but because such is the nature of the universe. These relations obtain whether any one perceives them or not, and so it is error to think or speak as if the existence or non-existence, in fact, of certain relations were conditioned by what we may do or omit, whether our conduct be physical or mental. In short, and ignoring for this turn the deeper truth that for the behests of science or philosophy relations are the very "stuff" of objects, we may say that by relations the universe of things is knit and re-knit, again and again, in all sorts of ways into a unitary whole, of which the details mutually explain and illustrate each other.

I have said enough, I think, to inform M. Mouret that the logic, science, and philosophy that I favor is throughout "objective" in its nature. M. Mouret will, I trust, pardon me, but it appears to me that he is biassed by a certain phobia he has towards things metaphysical, subjective, introspective, and *a priori*. I am as much the adversary of these when they are uncomprehended or miscomprehended and hence ill employed, as he can possibly be, but I conceive the fault to be not so much those things themselves as a certain spirit, attitude, and method that goes along with the lack of orientation with regard to them. Not so much metaphysics as metaphysicism is the bane of science and philosophy. While we are denouncing all tolerance of "metaphysical entities" some one may ask us what kind of entities are numbers, orders, forces, and the whole brood of entities, complex and less complex, among which mathematics makes its home.

As for *a priori* certitude, the mathematician should be the very last one to dis-

parage its validity or value. It is simple enough in its nature and it results from
the truth that the universe is interknit together by a network of relations that taken
by certain details are "formal," that is, are "forms," which forms and their respec-
tive components are in general susceptible of various definitions. We know that
one and one make two for the simple reason that two is no other than what one and
one make. We know that two and two make four of absolute necessity at all times,
in all places, and under all circumstances, because that which we call four has and
can have no other existence than as a member of a certain scale of numbers, formed
according to the rule we use in forming the same, and that, according to that rule,
which is a rule of operation, four is no other than the result which ensues from
taking one and one and one and one together, that is, this operation, or compound
of operations, has a certain result as related to the formation of the scale of num-
bers, and that result is identical with the result that ensues from the doubly com-
pound operation of forming first one two, and then another two, and then lastly
taking these two single twos together, and reading the result and giving it its name,
according to the names we have before given to our primary scale.

When we make such an affirmation as that two and two make four, we at bot-
tom asseverate, not with respect to objects, but with respect to our own mental
operations; in the case stated with respect to the equivalence of the results of cer-
tain complexes of our own mental operations. Now, to say, or, with Stuart Mill, to
propound, that at some epoch of time, or at some region of space, two and two
might make three or five, is not merely to say or propound that at such time or
place mental operation might work otherwise than it now does, for that might be,
and still consistent results ensue. It is no less than to say or propound that then
and there mental operation might exist that would be *inconsistent* in its results, that
is, that would be confounding, which is again only to say, that such mental opera-
tion would give no *mental* result at all.

A priori certainty, rightly regarded, only involves the faith that the universe is
a consistent one, a *reasonable* one; that while, as our experience, or our insight, or
our presumptions, may influence us, we may accept or reject, without any logical
sin, divers single doctrines, our liberty in general is limited, and we may not accept
or reject them by pairs and in other arrays, but *must* often, out of such arrays, hold
some doctrines as true and others as false, if we hold any as true or any as false.
We cannot, say, hold a triangle to be both scalene and equiangular, nor the circum-
ference of a circle to be just triple its diameter. When, being unsound as to our
fundamental notions and doctrines, we yet insist on their truth by authority of that
naïve intuition that is so unreliable, or by the authority of an incompetent logical
faculty, or slovenly logical habits, it is not any *a priori* assurance that we manifest,
it is only that false conceit that metaphysicism so generally fosters.

M. Mouret, in spite of his strong lucidity as to some very important points, and
in spite of his frequent protests, has not, as it seems to me, wholly escaped from
the meshes of metaphysicism. Not to mention other signs of this, his frequent use of

the notion "attribute" calls for notice. "Attribute" is the correlative of "substance" and has no proper sense apart from its correlative. It was the chief intent and import of the work of Berkeley and Hume, on whom M. Mouret so well relies, to abolish this notion of substance. We do not use either of these correlatives, unless it may be by inadvertence or in cases where there is no special call for precision, out of a desire to conform to the language understood by those whom we address. In our fashion of philosophising, one "thing" stands on just the same footing as another. They are all just so many mere "things" of equal original rank.

That "objective and abstract science" of "the sum total of the exterior objects of knowledge considered independent of their particular nature" is Relation-Lore, and because it was this science that I supposed M. Mouret at bottom to intend as his ideal of logic, I entitled my paper "Logic as Relation-Lore."

Since things, merely as so many somethings and nothing more, are the ultimate products of analysis, the science thereof, being the science of everything, (in virtue that everything must be at least a thing,) must be the true *scientia scientarum*, applying in its proper scope, without exception, to everything that has been, or is now, or may possibly be.

It is, in the first place, to be observed that this science is mathematical. It not merely contains the sciences of number and order. The sciences of number and then of order lie at its roots. Pythagoras is said to have laid it down that "number was the first principle of all things." We should be glad to ascertain that that great master had in mind the science that M. Mouret forecasts as his ideal of logic. Order may be "heaven's first law," but in science number is prior to order. Such science of order as is extant takes its very nomenclature from the designations of number.

But, contemporaneous with number and order, relation-lore obtains, and all the essential characteristics of relations are here to be observed. If in such fundamental branches of science as those of number and order that notion of a relation that knows the same as appurtenant to a particular term and not as a betweenness, is imperatively required, then we may be sure that that same notion must universally obtain. This question is easily settled. Take the numerical relation of A to B; say that relation is two. What is it that is two? Is it anything else than A? Is it anything between A and B? In the same case B is in relation to A, that is, it is one-half (not two) of A. What is it that is one-half, if not B, and B alone without any betweenness? What is it that is one-half but B, and what is it that is two but A?

Say again, that the relation of A to B is a relation of order, that is, say that B is third in order to A. So stated, the relation is not quite determinate, for A is also in the relation third in order to B, and each is in the relation *from* the other. Yet it is quite plain that A stands to B otherwise than B stands to A. We mark our feeling of this difference by saying that B is third in order *after* A, and that A is third in order *before* B. Now, what is it that is *after* A but B, and what is it that is

before B but *A* ? Caution is requisite here, because, though not involved, or, at least, involved only in an incidental way, there is in reality a betweenness, viz.: the thing that occupies the order second. Then, too, if we were dealing with the orders first and second, instead of the orders first and third, our natural associations would tend to drag in the interval or quasi-interval that is usually present in any concrete instance of order. We can escape this error by reflecting that in the latter case the order, in the abstract consideration, is in no wise dependent upon this interval or quasi-interval. We could just as well take this interval or quasi-interval as itself one of the terms of the order. The relations would be the same in either case.

If that which is named *after* is between *A* and *B*, then by that same token that which is named *before* is there also, in which case there is this dilemma : Either the relation "between" *A* and *B* is compounded of contraries, or there are the two relations, *after* and *before*, one pertaining to *B* and the other to *A*.

I respectfully submit that when M. Mouret on proceeding to study the relation of inequality finds himself compelled to give two *senses* to his betweennesses, he virtually yields the whole matter now in issue. Since a *sense*, in the meaning of M. Mouret, is the result of a more ultimate analysis than is a relation, (as he understands the same,) and since all relations, (as he conceives them,) may be regarded either as *senses*, or as compounded of senses, it is hard to see any good reason for his habitudes in respect to the nature of those important entities.

III. M. MOURET'S THEORY OF RELATIONS.

If I have justified the doctrine that I hold with respect to the nature of relations, it follows as a matter of course that M. Mouret labors under disadvantage in framing his theory thereof.

He tells us that his theory takes its true prompting and instruction, not from the axiom of symmetry, as I had supposed, but from the principle of Spencer. I can urge this in excuse, that of all the several maxims akin to it, the axiom of symmetry is the only one that bears any fruitful meaning. M. Mouret, very properly, as I think, insists on the constant recognition of the necessary relativity that must obtain as to all the objects of knowledge. He will have already seen by the foregoing that no one ought more strenuously to insist on such recognition than myself. Indeed, both he and myself are, by our fundamental doctrines, committed to the recognition that all the objects of knowledge are and must be interrelated ; that any such case as that of any object of knowledge unrelated is wholly inadmissible, and this naturally, insuperably, and *unconditionally*. We do not evoke the relations by our conduct ; they are there *pro re nata*. Now, I hardly know what to understand by an indefinite or by a constant relation. I can easily read into the maxims of Mr. Spencer and George Eliot, in each case, more than one intent, but for no intent that occurs to me can I perceive that either of these maxims are made of any considerable avail. Any two things have by the general case that allows of rela-

tivity at all a definite relation, in this, that they either coexist or they non-coexist, (one case of which non-coexistence is sequence). Just what rôle the definiteness or indefiniteness of the relations of either or both of them to a third thing may fulfil, either in supporting or in ascertaining either one of these definite relations, it is difficult for me to see. With regard to constancy; among all the relations that two things might bear to each other, it would be a singular case that would find them without any constant relation at all, so that again the rôle that the constancy or the inconstancy of the relations of either or both of them to a third thing might fulfil, either in supporting or in ascertaining the existence of an *unassigned* constant relation, is beyond my ability to state. Hence, to lay it down with gravity that "things that have a definite relation to the same thing have a definite relation to one another,'' or that "things that have a constant relation to the same thing have a constant relation to one another," is only to imply a dependence upon or a contingency upon that which in reality is altogether lacking, so far as we can see, in governing or in instructive efficacy.

The truth is, that after we have recognised the subsistence of universal relativity, it is precisely the ascertaining of the different kinds of relations and the ascertaining of the connexions of these different *kinds* of relations with one another that can alone benefit us to any considerable extent. This is not, as M. Mouret supposes, a matter of psychology, an ascertaining of that which constitutes "the subjective element," but an "objective and abstract" study of "the sum total of the exterior objects of knowledge," and at least in the earlier stages of this study it is very much facilitated by considering them "independent of their particular nature." Hence I must decidedly disapprove of the method of M. Mouret, in selecting a lot of concrete examples for study. It seems to me a useless and needless invitation to error.

I have no special fault to find with the three leading principles laid down by M. Mouret with respect to concepts, save, perhaps, in so far as he claims that the negative of a concept is by any necessity single. I must protest, however, that I cannot agree with him that these three leading principles "cover the entire subject." I do not even see that they are, in essence, new. He may have shown what is undoubtedly true, that many of the concepts of scientific order are composed of less complex elements, but I cannot assent to the proposition that these less complex elements, or even that the most simple thereof, are reducible to "states of consciousness" until I know better what is to be understood by the phrase "states of consciousness"; a phrase, I may add, that is very much overworked. A "state of consciousness" may be taken to mean the "form" expressed therein, or it may mean the various psychological effects, which, as I have before stated, I consider in themselves as both irreducible and valueless for the behests of both science and philosophy. In either case the solution claimed by M. Mouret is obnoxious to the criticism that consciousness grades off continuously into unconsciousness, and, although we may lay it down that every notion or relation ought under analysis of

adequate power to resolve without limit into other notions or relations, we may not say of any status that just emerges over the threshold of consciousness that its elements are primordial. Contrariwise we should expect and hold that our most simple notions and relations depend upon components that are not perceptible or that are sub-perceptible only. All this, however, is the instruction that the introspective method yields. The "objective and abstract" science above mentioned involves no such problems. I must also record my respectful dissent from the claim of M. Mouret that he has pointed out the order and conditions of the derivation of concepts, and that he has described all the conditions for the passage from the known to the unknown.

IV. MATHEMATICAL EQUALITY.

M. Mouret says in his foot-note to section (2) of the paper I criticised : "In the present essay I use the word mathematics to signify exclusively the science of numbers and of quantities, in technical terms the theory of numbers and of mathematical analysis," and his context shows that he expressly excluded geometry, mechanics, physics, chemistry, etc. Hence I took him to mean by "mathematical equality" numerical equality, at least in the main. The gist of what he says upon this branch of our subject in his reply may be comprised in his remark, that equality and inequality are correlative relations, which is entirely true, and in his claim that equality is logically prior to inequality, inequality being defined by equality. In my own paper I advanced an argument designed expressly to show that this was not the case, equality being defined by two correlative inequalities. I can only reiterate my former argument, which as yet remains unanswered. I may say here that I believe I am in accord with many digtinguished logicians in holding that the only propositions that are unconditional in their signification are and must be negative in their form. The remark of Hegel that all determination is by negation is, I believe, well approved as a logical principle.

V. THE AXIOM OF SYMMETRY.

I cannot see that M. Mouret has justified the axiom of symmetry or in any wise parried the thrusts I gave it. Most of his arguments depend upon the validity of his view of the nature of relations. That axiom is stated in an unqualified way, and it asserts no less than that whenever or wherever, no matter how brief the instant or how contracted the region, any two things have the same symmetrical relation to a third thing, that then and there that same symmetrical relation exists between the first two things. Clearly, then, as it seems to me, it was only open to M. Mouret to show as to the instances that I cited, either that the axiom was fulfilled or that my instances were *not* cases of the conjoint existence of a like symmetrical relation between each of two things to a third thing. Now, what kind of an axiom is that, that when the case that it contemplates exists, is sometimes true and sometimes not true ? The case of mutual friendship supposes that A and C are mutual friends, coincidently that B and C are also mutual friends. Suppose that this

state of affairs endures for an instant only, then if mutual friendship is a symmetrical relation, and if the axiom is valid, A and B must be mutual friends. I appealed to experience that such was often not the result, and while much that M. Mouret says in avoidance is very true, it does not as it seems to me at all meet the exigency of the case which is very simple, viz.: Does mutual friendship exist between A and B if it exists between A and C and at the same time between B and C? or on the other hand, Is mutual friendship a symmetrical relation?

The case of the distance of two points from a third is made totally irrelevant owing to the different views held by M. Mouret and myself as to the nature of relations. He looks upon a relation as a betweenness, and consequently a distance is to him only a single relation, while I regard the same as the conjunction of two convertible relations.

The case of the sun and two planets is avoided by M. Mouret by the denial that there is in that case any conjunction of relations of mutual equilibrium. He sees in that case only relations of mutual attraction. I cannot stop to dispute over the question of equilibrium. It is a relation that would require much time and space to demonstrate as existing in the case in question in every scientific sense. Instead of that I will take the case of a similar kind to that admitted by M. Mouret; viz., a case of mutual attraction. A and B are bodies charged with positive electricity, and C is a body charged with negative electricity. Hence between A and C there is a mutual attraction, and so is there also between B and C, but instead of A and B attracting one another as by the axiom of symmetry they should, they on the very contrary repel one another. The simple objective verity does not hold.

In order that M. Mouret may not think that I have disregarded his "axiom of the three senses" and what he has to say on its behalf, I make this mention. I must protest that this new axiom is in just as bad a plight as is the axiom of symmetry, but the already too great length of this paper forbids me to enter upon any discussion of the same.

In conclusion I wish to thank M. Mouret for his notice of my criticisms. If in the ardor of advocacy I have been betrayed into any remark that seems to him ungracious, I wish to blot it out and to assure him not formally but really of my distinguished regard.

FRANCIS C. RUSSELL.